MATHEMATICS

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AIM POINTS
MATHEMATICS
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FUNCTIONS

& Their Properties

CONTENTS

THINGS TO REMEMBER

1. Let A and B be two non–empty sets. Then, a subset f of A × B is a function from A to B, if
   (i) for each a A there exists $b \in B$ such that $(a, \in b) \in f$.
   (ii) $(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$
   In other words, a subset f of A × B is a function from A to B, if each element of A appears in some ordered pair in f and no two ordered pairs in f have the same first element.
2. Let A and B be two non–empty sets. Then, a function f from A to B associates every element of A to a unique element of B. The set A is called the domain of f and the set B is known as its co–domain. The set of images of elements of set A is known as the range of f.
3. If $f : A \to B$ is a function, then
   $x = y \Rightarrow f(x) = f(y)$ for all $x, y \in A$.
4. A function $f : A \to B$ is a one–one function or an injection, if
   $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$
   or, $x \neq y \Rightarrow f(x) \neq f(y)$ for all $x, y \in A$.
5. A function $f : A \to B$ is an onto function or a surjection, if range (f) = co–domain (f).
6. Let A and B be two finite sets and $f : A \to B$ be a function.
   (i) If f is an injection, then $n(A) \leq n(B)$
   (ii) If f is a surjection, then $n(A) \geq n(B)$
   (iii) If f is a bijection, then $n(A) = n(B)$.
7. If A and B be two finite sets and $f : A \to B$ be a function.
   (i) Number of functions from A to B = n^m.
   (ii) Number of one–one functions from A to B $= \begin{cases} {}^nC_m \times m f, \text{if}, n \geq m \\ 0, \text{if } n < m \end{cases}$
   (iii) Number of onto functions from A to B $= \begin{cases} \sum_{T=1}^{n} (-1)^{n-r} \; {}^nC_m \times m f, \text{if}, n \geq m \\ 0, \text{if } m < n \end{cases}$
   (iv) Number of one–one and onto functions from A to B $= \begin{cases} n f, \text{if } m = n \\ 0, \text{if } m \neq n \end{cases}$
8. If a function $f : A \to B$ is not an onto function, then $f : A \to f(A)$ is always an onto function.
9. The composition of two bijections is a bijection.
10. If $f : A \to B$ is a bijection, then $g : B \to A$ is inverse of f, iff
    $f(x) = y \Leftrightarrow g(y) = x$
    or, $gof = I_A$ and $fog = I_B$.
11. Let $f : A \to B$ and $g : b \to A$ be two functions.
    (i) If $gof = I_A$ and f is an injection, then g is a surjection.
    (ii) If $fog = I_B$ and f is a surjection, then g is an injection.
12. Let $f : A \to B$ and $g : B \to C$ be two functions. Then
    (i) $gof : A \to C$ is onto $\Rightarrow g : B \to C$ is onto.
    (ii) $gof : A \to C$ is one–one $\Rightarrow g : A \to B$ is one–one

(iii) gof : $A \to C$ is onto and g : $B \to C$ is one–one $\Rightarrow$ f : $A \to B$ is onto.
(iv) gof : $A \to C$ is one–one and f : $A \to C$ is onto $\Rightarrow$ f : $B \to C$ is one–one.

EXERCISE-1

1. Let A and B be two non–empty sets. A relation f from A to B i.e., a sub set of $A \times B$ is called a function (or a mapping or a map) from A to B, if
   (i) for each $a \in f$ A there exists $b \in B$ such that $(a, b) \in f$
   (ii) $(a, b) \in f$ and $(a, c) \in f \Rightarrow b = c$.
2. Let A and B be two non–empty sets. Then a function. 'f' from set A to set B is a rule or method or correspondence which associates elements of set A to elements of set B such that :
   (i) all elements of set A are associated to elements in set B.
   (ii) an element of set A is associated to a unique element in set B.

   In other words, a function 'f' from a set A to a set B associates each element of set A to a unique element of B.
3. If k is a fixed real number, then a function f(x) given by
   $f(x) = k$ for all $x \in R$
   is called a constant function.
4. The function that associated each real number to itself is called the identity function and is usually denoted by I.
   Thus, the function I : $R \to R$ defined by
   $I(x) = x$ for all $x \in R$
   is called the identity function.
5. The function f(x) defined by
   $$f(x) = |x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$$
   is called the modulus function.
6. For any real number x, we use the symbol [x] or, $\lfloor x \rfloor$ to denote the greatest integer less than or equal to x. For example,
   $[2.75] = 2$, [3], $[0.74] = 0$, $[-7.45] = -8$ etc.
   The function f : $R \to R$ defined by
   $f(x) = [x]$ for all $x \in R$
   is called the greatest integer function or the floor function.
7. If n is an integer and x is a real number between n and n + 1, then
   (i) $[-n] = -[n]$
   (ii) $[x + k] = [x] + k$ for any integer k.
   (iii) $[-x] = -[x] - 1$
   (iv) $$[x] + [-x] = \begin{cases} -1, & \text{if } x \notin Z \\ 0, & \text{if } x \in Z \end{cases}$$

(v) $[x] - [-x] = \begin{cases} 2[x]+1, & \text{if } x \notin Z \\ 2[x], & \text{if } x \in Z \end{cases}$

(vi) $[x] \geq k \Rightarrow x \geq k$, where $k \in Z$

(vii) $[x] \leq k \Rightarrow x < k + 1$, where $k \in Z$

(viii) $[x] > k \Rightarrow x \geq k + 1$, where $k \in Z$

(ix) $[x] < k \Rightarrow x < k$, where $k \in Z$

(x) $[x + y] = [x] + [y + x - [x]]$ for all $x, y \in R$

(xi) $[x]+\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right]+\ldots+\left[x+\frac{n-1}{n}\right]=[nx], n \in N$

8. For any real number x we use the symbol $\lceil x \rceil$ to the smallest integer than or equal to x.
For example,

$\lceil 4.7 \rceil = 5, \lceil -7.2 \rceil = -7, \lceil 5 \rceil = 5, \lceil 0.75 \rceil = 1$ etc.

The function $f : R \to R$ defined by

$f(x) = \lceil x \rceil$ for all $x \in R$

is called the smallest integer function or the ceiling function.

9. Following are some properties of smallest integer function :

(i) $\lceil -n \rceil = -\lceil n \rceil$, where $n \in Z$

(ii) $\lceil -X \rceil = -\lceil x \rceil + 1$, where $x \in R - Z$

(iii) $\lceil x + n \rceil = \lceil x \rceil + n$, where $x \in R - Z$ and $n \in Z$

(iv) $\lceil x \rceil = \lceil \square - x \rceil = \begin{cases} -1, & \text{if } x \notin Z \\ 0, & \text{if } x \in Z \end{cases}$

(v) $\lceil x \rceil + \lceil -x \rceil = \begin{cases} 2\lceil x \rceil - 1, & \text{if } x \notin Z \\ 2\lceil x \rceil, & \text{if } x \in Z \end{cases}$

10. For any real number x, we use the symbol $\{x\}$ to denote the fractional part or decimal part of x. For example,

$[3.45] = 0.45, [-2.75] = 0.25, [-0.55] = 0.45, [3] = 0, [-7] = 0$ etc.

The function $f : R \to R$ defined by

$f(x) = \{x\}$ for all $x \in R$

is called the fractional part function.

11. The function f defined by

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

or, $$f(x)=\begin{cases}1, & x>0\\ 0, & x=0\\ -1, & x=0\end{cases}$$

is called the signum function.

12. If a is a positive real number other than unity, then a function that associates each $x \in R$ to a^x is called the exponential function.
13. If a > 0 and a ≠ 1, then the function defined by

    $f(x) = \log_a x, x > 0$

    is called the logarithmic function.
14. The function $f : R^+ \to R$ defined by

    $f(x) \ +\sqrt{x}$

    is called the square root function.
15. The function $f : R \to R$ defined by

    $f(x) = x^2$
16. The function $f : R \to R$ defined by

    $f(x) = x^3$
17. The function $f : R \to R$ defined by

    $f(x) = x^{1/3}$
18. Let $f : D \to R$ ba a real function and α be a scalar (real number). Then the product α f is a function from D to R and is defined as

    $(\alpha f)(x) = \alpha f(x)$ for all $x \in D$.
19. Let A = {1, 2, 3}, B = {4, 5, 6, 7} and let f = {(1, 4), (2, 5), (3, 6)} be a function from A to B. Then, f(1) = 4, f(2) = 5 and f(3) = 6. Clearly, different elements of A have different images in B. So, f is a one–one function.
20. Show that the function $f : Z \to Z$ defined by $f(x) = x^2 + x$ or all $x \in Z$, is a many–one function.
21. Show that the function $f : N \to N$ given by f(1) = f(2) = 1 and f(x) = x – 1 for every x ≥ 2, is onto but not one–one.
22. Prove that $f : R \to R$, given by f(x) = 2x, is one–one and onto.
23. Show that the function $R \to R$, defined as $f(x) = x^2$, is neither one–one nor onto.
24. Show that $f : R \to R$, defined as $f(x) = x^3$, is a bijection.
25. Show that the function $f : R_0 \to R_0$, defined as $f(x) = \frac{1}{x}$, is one–one onto, where R_0 is the set of all non–zero real numbers. Is the result true, if the domain R_0 is replaced by N with co–domain being same as R_0 ?
26. Prove that the greatest integer function $f : R \to R$, given by f(x) = [x], is neither one–one nor onto, where [x] denotes the greatest integer less than or equal to x
27. Show that the modulus function $f : R \to R$, given by f(x) = | x | is neither one–one nor onto.

28. Show that the function $f : R \to R$ given by $f(x) = ax + b$, where $a, b \in R$, $a \neq 0$ is a bijection.

29. Let $A = R - \{2\}$ and $B = r - \{1\}$. If $f : A \to B$ is a mapping defined by $f(x) = \frac{x-1}{x-2}$, show that f is bijective.

30. Let A and B be two sets. Show that $f : A \times B \to B \times A$ defined by $f(a, b) = (b, a)$ is a bijection.

31. Consider the identity function $I_N : N \to N$ defined as

$I_N(x) = x$ for all $x \in N$

Show that although I_N is onto but $I_N + I_N : N \to N$defined as

$(I_N + I_N)(x) = I_N(x) + I_N(x) = x + x = 2x$

is not onto.

32. Let $f : X \to Y$ be a function. Define a relation R on X given by

$R = \{(a, b) : f(a) = f(b)\}$.

Show that R is an equivalence relation on X.

33. Classify the following functions as injection, surjection or bijection :

(i) $f : N \to N$ given by $f(x) = x^2$

(ii) $f : Z \to Z$ given by $f(x) = x^2$

(iii) $f : N \to N$ given by $f(x) = x^3$

(iv) $f : Z \to Z$ given by $f(x) = x^3$

(v) $f : R \to R$ defined by $f(x) = |x|$

(vi) $f : Z \to Z$ defined by $f(x) = x^2 + x$

(vii) $f : Z \to Z$ defined by $f(x) = x - 5$

(viii) $f : R \to R$, defined by $f(x) = \sin x$

(ix $f : R \to R$, defined by $f(x) = x^3 + 1$

(x) $f : R \to R$, defined by $f(x) = x^3 - x$

(xi) $f : R \to R$, defined by $f(x) = \sin^2 x + \cos^2 x$

(xii) $f : Q - \{3\} \to Q$, defined by $f(x) = \frac{2x+3}{x-3}$

(xiii) $f : Q \to Q$, defined by $f(x) = x^3 + 1$

(xiv) $f : R \to R$, defined by $f(x) = 5x^3 + 4$

(xv) $f : R \to R$, defined by $f(x) = 3 - 4x$

(xvi) $f : R \to R$, defined by $f(x) = 1 + x^2$

34. Show that the function $f : R - (3) \to R - (1)$ given by $f(x) = \frac{x-2}{x-3}$

35. Show that $f : R \to R$, given by $f(x) = x - [x]$, is neither one–one nor onto.

36. If $A = \{1, 2, 3\}$, show that a one–one function $f : A \to A$ must be onto.

37. If $A = \{1, 2, 3\}$, show that a onto function $f : A \to A$ must be one–one.

38. Find the number of all onto functions from the set A = {1, 2, 3, ..., n} to itself.
39. Let $f : R \to R$; $f(x) = \sin x$ and $g : R \to R$; $g(x) = x^2$ find fog and gof.
40. Let f : {2, 3, 4, 5} → {3, 4, 5, 9} and g : {3, 4, 5, 9) → {7, 11, 15} be functions defined as

    f(2) = 3, f(3) = 4, f(4) = f(5) = 5

    and g(3) = g(4) = 7 and g(5) = g(9) = 11

    Find gof.
41. Let f : {1, 3, 4} → {1, 2, 5} and g : {1, 2, 5} → {1, 3} be given by f = {(1, 2), (3, 5), (4, 1)} and g = {(1, 3), (2, 3), (5, 1)}. Write down gof.
42. Find gof and fog, if f : R → Rand g : R → R are given by $f(x) = |x|$ and $g(x) = |5x - 2|$.
43. If the functions f and g are given by f = {(1, 2), 3, 5), (4, 1)} and g = {(2, 3), (5, 1), (1, 3)}, find range of f and g. Also write down fog and gof as sets of ordered pairs.
44. If the function f : R → R be given by $f(x) = x^2 + 2$ and g : R → R be given by $g(x) = \frac{x}{x-1}$. Find fog and gof.
45. If $f : R - \left\{\frac{7}{5}\right\} \to R - \left\{\frac{3}{5}\right\}$ be defined as $f(x) = \frac{3x+4}{5x-7}$ and $g : R - \left\{\frac{3}{5}\right\} \to R - \left\{\frac{7}{5}\right\}$ be defined as $g(x) = \frac{7x+4}{5x-3}$. Show that gof = I_A and fog = I_B, where $B = R - \left\{\frac{3}{5}\right\}$ and $A = R - \left\{\frac{7}{5}\right\}$.
46. If f : R → R is defined by $f(x) = x^2 - 3x + 2$, find f(f(x)).
47. Let $A = \{x \in R : 0 \le x \le 1\}$. If f : A → R is defined by

    $$f(x) = \begin{cases} x, \text{if} x \in Q \\ 1 - x, \text{if } x \notin Q \end{cases}$$

    then prove that fof (x) = x for all $x \in A$.
48. Let f : R → R and g : R → R be two functions such that $fog(x) = \sin x^2$ and $gof(x) = \sin^2 x$. Then, find f(x) = g(x).
49. Let f : Z → Z be defined by f(n) = 3n for all $n \in Z$ and g : Z → Z e defined by

    $$g(n) = \begin{cases} \frac{n}{3}, \text{if n is a multiple of 3} \\ 0, \text{if n is not a multiple of 3} \end{cases} \quad \text{for all } n \in Z$$

    Show that gof = I_Z and fog ≠ I_Z.
50. Let f, g and h be functions from R to R. Show that :
    (i) (f + g) oh = foh + goh
    (ii) (fg) oh = (foh) (goh)

51. Let f : R → R be the signum function defined as $f(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$

and g : R → R be the greatest integer function given by g(x) = [x]. Then, prove that fog and gof coincide in [–1, 0).

52. The composition of functions is not commutative i.e. fog ≠ gof.

53. The compsition of functions is associative i.e. if f, g, h are three functions such that (fog)oh and fo(goh) exist, then

(fog) oh = fo (goh)

54. The composition of two bijections is a bijection i.e. if f and g are two bijections, then gof is also a bijection.

55. Let f : A → B, g : B → A be two functions such that gof = I_A. Then, f is an injection and g is a surjection.

56. Let f : A → B and g : B → A be two functions such that fog = I_B. Then, f is a surjection and g is an injection.

57. Consider f : N → N, g : N → N and h : N → R defined as f(x) = 2x, g(y) = 3y + 4 and h(z) = sin z for all x, y, z ∈ N. Show that ho (gof) = (hog) of.

58. Give examples of two functions f : N → Z nd g : Z → Z such that gof is injective but g is not injective.

59. If f : R → R and g : R → R be functions defined by

$f(x) = x^2 + 1$ and g(x) = sin x

then find fog nd gof.

60. Uf $f(x) = e^x$ and $g(x) = \log_e x$ (x > 0), find fog and gof. Is fog = gof ?

61. If f(x) = $f(x) = \sqrt{x} (x \geq 0)$ nd $g(x) = x^2 - 1$ are two real functions, find fog and gof. Is fog = gof ?

62. If $f(x) = \frac{1}{x}$ and g(x) = 0 are two real functions, show that fog is not defined.

63. Let f(x) = [x] and g(x) = | x |. Find

(i) $(gof)\left(\frac{-5}{3}\right) - (fog)\left(\frac{-5}{3}\right)$

(ii) $(gof)\left(\frac{5}{3}\right) - (fog)\left(\frac{5}{3}\right)$

(iii) (f + 2g) (–1)

64. Let $f(f(x)) = \frac{2x+1}{2x+3}$ for all x ∈ R, $x \neq -\frac{1}{2}, -\frac{3}{2}$

65. Let f be a real function defined by $f(x) = \sqrt{x-1}$. Find (fofof) (x).

Also, show that fof ≠ f^2.

66. If $f(x) = \frac{x-1}{x+1}, x \neq -1$, then show that $f(f(x)x) = -\frac{1}{x}$ provided that x ≠ 0, –1.
67. Let f be any real function and let g be a function given by g(x) = 2x. Prove that gof = f + f.
68. If the function f : R → R be defined by $f(x) = x^2 + 5x + 9$, find $f^{-1}(8)$ and $f^{-1}(9)$.
69. Let f : R → R be defined as $f(x)\ x^2 + 1$. Find
    (i) $f^{-1}(-5)$ (ii) $f^{-1}(26)$ (iii) $f^{-1}\{10, 37\}$
70. Let S = {1,2,3}. Determine whether the function f : S → S defined as below have inverse. Find f^{-1}, if it exists.
    (i) f = (1, 1), (2, 2), (3, 3)
    (ii) f = {(1, 2), (2, 1), (3, 1)
    (iii) f = {(1, 3), (3, 2), (2, 1)}.
71. Consider f : {1, 2,3} → {a, b, c} given by f(1) = a, f(2) = b and f(3) = c. Find the inverse $(f^{-1})^{-1}$ of f^{-1}. Show that $(f^{-1})^{-1} = f$.
72. Let f : R → R be defined by f(x) = 3x – 7. Show that f is invertible and hence find f^{-1}.
73. Show that f : R – {0} → R – {0} given by $f(x) = \frac{3}{x}$ is invertible and it is inverse of itself.
74. Let f : N ∪ {0} → N ∪ (0) be defined by

$$f(n) = \begin{cases} n+1, \text{if } n \text{ is even} \\ n-1, \text{if } n \text{ is odd} \end{cases}$$

Show that f is invertible and $f = f^{-1}$.

75. The inverse of a bijection is unique.
76. The inverse of a bijection is also a bijection.
77. If f : A → B is a bijection and g : B → A is the inverse of f, then fog = I_B and gof = I_A, where I_A and I_B are identity functions on the sets A and B respectively.
78. If f : A → B and g : B → C are two bijections, then gof : A → C is bijection and $(gof)^{-1} o g^{-1}$.
79. Let f : A → B and g : B → A be two functions such that gof = I_A and fog = I_B. Then, f and g are bijections and $g = f^{-1}$.
80. Let f : A → B be an invertible function. Show that the inverse of f^{-1} is f. i.e., $(f^{-1})^{-1} = f$.
81. Prove that the function f : R → R defined as f(x) = 2x – is invertible. Also, find f^{-1}.
82. Show that the function f : R → R is given by $f(x) = x^2 + 1$ is not invertible.
83. If the function f : [1, ∞) defined by $f(x) = 2^{x(x-1)}$ is invertible, find $f^{-1}(x)$.
84. Find the value of parameter α for which the function f(x) = 1 + α x, α ≠ 0 is the inverse of itself.
85. Let f : N → Y be a function defined of f(x) = 4x + 3, where
    Y : {y ∈ N : y = 4x + 3 for some x ∈ N}. Show that is invertible. Find its inverse.
86. Let $Y = \{n^2 : n \in N\} \subset N$. Consider f : N → Y given by $f(n) = n^2$. Show that f is invertible. Find the inverse of f.

87. Let f : N → R be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that f : N → Range (f) is invertible. Find the inverse of f.

88. Show that f : [–1, 1] → R, given by $f(x) = \frac{x}{x+2}$ is one–one. Find the inverse of the function f : [–1, 1] → Range (f).

89. Range (f) : Let f : R → R be defined as f(x) = 10x + 7. Find the function g : R → R such that gof = fog = I_R.

90. Find f^{-1} if it exists : f : A → B where
    (i) A = {0, –1, –3, 2}; B = {–9, –3, 0, 6} and f(x) = 3x.
    (ii) A = {1, 3, 5, 7, 9}; B = (0, 1, 9, 25, 49, 81} and $f(x) = x^2$.

91. Let A = {1, 2, 3, 4}; B = {3, 5, 7, 9}, C = {7, 23, 47, 79} and f : A → B, g : B → C be defined as f(x) = 2x + 1 and $g(x) = x^2 - 2$. Express $(gof)^{-1}$ and f^{-1} og^{-1} as the sets of ordered pairs and verify that $(gof)^{-1} = f^{-1}$ og^{-1}.

92. A function f : R → R is defined as $f(x) = x^3 + 4$. Is it a bijection or not ? In case it is a bijection, find f^{-1} (3).

93. If $f(x) = \frac{4x+3}{6x-4}, x \neq \frac{2}{3}$, show that fof (x) = x for all $x \neq \frac{2}{3}$. What is the inverse of f ?

94. Consider f : R → R given by f(x) = 4x + 3. Show that f is invertible. Find the inverse of f.

95. Consider f : R → R_+ → [4, ∞) given by f(x) $x^2 + 4$. Show that f is invertible with inverse f^{-1} of f given by $f^{-1}(x) = \sqrt{x-4}$, where R^+ is the set of all non–negative real numbers.

96. Consider f : R_+ → [–5, ∞) given by f(x) $9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(x) = \frac{\sqrt{x+6}-1}{3}$.

97. Consider f : {1, 2, 3} → {a, b, c} and g : {a, b, c} → {apple, ball, cat} defined as f(1) = a, f(2) = b ,
    f(3) = c, g(a) = apple, g(b) = ball and g(c) = cat. Show that f, g and gof are invertible. Find f^{-1}, g^{-1} and show that $(gof)^{-1} = f^{-1}o\ g^{-1}$.

98. Write total number of one–one functions from set A = {1, 2, 3, 4} to set B = {a, b, c}.

99. Let C denote the set of all complex numbers. A function f : C → C is defined by $f(x) = x^3$. Write $f^{-1}(1)$.

100. If f : C → C is defined by $f(x) = (x - 2)^3$, write f^{-1})–1)

101. If f : R → R defined by f(x) = 3x – 4 is invertible then write $f^{-1}(x)$.

102. Write the domain of the real function $f(x) = \frac{1}{\sqrt{|x|-x}}$.

103. Wha is the range of the function $f(x) = \frac{|x-1|}{x-1}$?

104. If f: R → R be defined by $f(x) = (3 - x^3)^{1/3}$, then find fof(x).

EXERCISE-3

1. Which of the following functions from A = {x : $-1 \le x \le 1$} to itself are bijections ?
   (a) $f(x) = \frac{x}{2}$ (b) $g(x) = \sin\left(\frac{\pi x}{2}\right)$ (c) $h(x) = |x|$ (d) $k(x) = x^2$

2. The function f : R → R defined by
   f(x) = (x – 1) (x – 2) (x – 3) is
   (a) one–one but not onto (b) onto but not one–one
   (c) both one and onto (d) neither one–one nor onto

3. Let f : R – {n} → R be a function defined by
   $f(x) = \frac{x-m}{x-n}$, where m ≠ n. Then
   (a) f is one–one onto (b) f is one–one into
   (c) f is many one onto (d) neither one–one nor onto

4. The function f : R → R, $f(x) = x^2$ is
   (a) injective but not surjective (b)surjective but not injective
   (c) injective as well as surjective (d) neither injective nor surjective

5. Which of the following functions from
   A = {x R : $-1 \le x \le 1$} to itself are bijections ?
   (a) $f(x) = |x|$ (b) $f(x) = \sin\frac{\pi x}{2}$
   (c) $f(x) = \sin\frac{\pi x}{4}$ (d) none of these

6. If g (f(x)) = |sin x| and $f(g(x)) = \left(\sin\sqrt{x}\right)^2$
   (a) $f(x) \sin^2 x$, $g(x) = \sqrt{x}$ (b) f(x) = sin x, g(x) = | x |
   (c) $f(x) = x^2$, $g(x) = \sin\sqrt{x}$ (d) f and g cannot be determined

7. Let A = {x ∈ R : x ≥ 1}. The inverse of the function f : A → A given by f(x) $2^{x(x-1)}$, is
   (a) $\left(\frac{1}{2}\right)^{x(x-1)}$ (b) $\frac{1}{2}\left\{1+\sqrt{1+4\log_2 x}\right\}$
   (c) $\frac{1}{2}\left\{1-\sqrt{1+4\log_2 x}\right\}$ (d) not defined

8. Let $f(x) = \frac{1}{1-x}$. Then |fo(fof)| (x)
   (a) x for all x ∈ R (b) x for all x ∈ R– {1}
   (c) x for all x ∈ R – |0, 1| (d) none of these

9. If F : [1, ∞) → [2, ∞) is given by $f(x) = x + \frac{1}{x}$, then $f^{-1}(x)$ equals.

(a) $\frac{x+\sqrt{x^2-4}}{2}$ (b) $\frac{x}{1+x^2}$

(c) $\frac{x-\sqrt{x^2-4}}{2}$ (d) $x+\sqrt{x^2-4}$

10. Lef f(x) = $\frac{\alpha x}{x+1}, x \neq 1$. Then, for what value of α is f(f(x)) = x ?

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d) –1

11. If f : R → (–1, 1) is defined by f(x) = $\frac{-x|x|}{1+x^2}$, then $f^{-1}(x)$ equals

(a) $\sqrt{\frac{|x|}{1-|x|}}$ (b) $-\text{sgn}(x)\sqrt{\frac{|x|}{1-|x|}}$

(c) $\sqrt{\frac{x}{1-x}}$ (d) none of these.

12. If f(x) = $\sin^2 x$ and the composite function g(f(x)) = | sin x |, then g(x) is equal to

(a) $\sqrt{x-1}$ (b) $\sqrt{x}$ (c) $\sqrt{x+1}$ (d) $-\sqrt{x}$

13. Let f : R → R be given by f(x) = $x^2 - 3$. Then, f^{-1} is given by

(a) $\sqrt{x+3}$ (b) $\sqrt{x}+3$ (c) $3+\sqrt{x}$ (d) none of these